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CHAPTER THIRTEEN

RESEARCH PROBLEMS

Why Learn This

In previous chapters you have explored a variety of techniques and strategies for solving many different types of problems, and you have learned how to use different tools in different particular situations. Most problems that we encounter in life, however, do not come pre-associated with a particular chapter in a book or method of solution, so a valuable skill to develop is to be able to judge which tools may be suited to tackling a new problem.

The problems in this chapter provide a way for you to validate your understanding of the Problem Solving Methodology and give you experience transferring that understanding to new situations. As you overcome the challenges presented here, your confidence in problem solving will grow, and the skills you have developed in this course will be more easily transferred to other courses, to the workplace, and to the rest of your life.

Orientation

The problems in this chapter are all thought-provoking, and most are open-ended, with no single correct solution. A few draw on the skills developed in a single chapter, while most draw upon several. Students and teams will need to think critically to identify all of the important considerations and to make appropriate assumptions. Some problems require library or other research to obtain necessary data. Many are suitable for term projects by a team, to culminate in a class presentation of the problem, its solution, and the processes used.

In this chapter there are no defined activities as such. You will need to make your own plan for solving each problem, determine and seek out any needed information, and define your own Critical Thinking Questions as you proceed. In this chapter, you are explorers on uncharted seas! Good luck!

Learning Objectives

1. To gain experience selecting appropriate tools for approaching a new task
2. To transfer skills gained in previous chapters to the solution of new types of problems
3. To validate and increase your understanding of principles of problem solving
4. To be able to develop an extended solution of a complex problem and make an effective and convincing presentation of your solution.

Performance Criteria

1. Appropriateness of the techniques used to solve each problem
 - Measure 1: Quality of your justification for using the techniques you used
 - Measure 2: Success of the techniques in solving the problem
2. Completeness and correctness of your solutions
 - Measure 1: Are all important factors taken into account?
 - Measure 2: Have you thoroughly validated your results?
3. Quality of the presentation of your results
 - Measure 1: Completeness: Is there sufficient detail of your methods and computation?
 - Measure 2: Clarity of presentation
 - Measure 3: Quality of supporting materials

Prerequisites

Each problem in this chapter has its own prerequisites. Part of the challenge in this chapter is in evaluating just what you need to know in order to solve each problem. For most of them, the following general list may be helpful:

1. A clear understanding of the Problem Solving Methodology
2. Fundamental understanding of spreadsheets, and/or word processors
3. The ability to explore and experiment and use your imagination
4. Mastery of the methodologies and techniques covered in several of the chapters of this book.

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1. THE CAR PROBLEM. You plan to travel 12,000 miles during each of the next four years and the average cost of gasoline will be \$1.20 per gallon. Which car should you buy?

	Car 1	Car 2
Price	\$9,000	\$11,000
MPG	24	32
Resale Price	\$3,500	\$ 4,000
Maintenance/Year	\$ 340	\$ 275
Insurance/Year	\$ 280	\$ 400

2. THE COMPUTER PROBLEM. Should you buy a personal computer? If so, what should you buy? Estimate how many term papers and problem sets you will be expected to complete during your college career. Obtain hardware and software costs from your local computer store.

3. FORCE TRANSDUCER. A force transducer is a device which converts a force input into a voltage signal. The following calibration data was obtained for a new transducer. Using graphical analysis, determine the sensitivity of this device (i.e., change in voltage output ÷ change in force input). Use regression analysis.

lbf	V
0.0	.05
0.5	.35
1.0	.73
2.0	1.52
5.0	3.42
10.0	7.08

4. ELEMENTARY SCIENCE PROJECT. City elementary school students, grades 1 - 6, are required to participate in the Science Fair. Each student is responsible for the design, execution, and report of a project. The school has limited resources and can only offer one first place, one second place, and one third place overall, with an Honorable Mention for the top project in each of the other grades.

Develop assessment criteria for the judges to use. Remember that the first grade student must have the same opportunity to win as the sixth grade student.

5. SCHOOL EXPENSES. Create a table of data: each column describes a year of schooling that you are planning. The rows of the table should be labeled to include your major expense categories; include items such as tuition, transportation, rent, food, books and supplies, entertainment, etc. Graph your total expenses over time. Store your table in an ASCII file. Submit a hard copy of this file.

6. CHECK TRAVELS. When a customer pays a store with a check, the check has to travel back from the store to the customer while cash travels from the customer's bank to the store.

List the steps needed to move the check from the store to the customer and a second list of the steps needed to move money from the customer's bank to the store.

- At each step of each list, list the most likely errors that could occur and the person, machine, or organization responsible for finding, fixing, and preventing such errors.
- For each error, design a data collection process which balances the desire for accurate check processing against the cost of processing checks and reducing errors.
- Assuming your data collection process has found the largest source of errors in the process and that you are the person responsible for reducing such errors, how would you do so?

7. LIFE EXPECTANCY. Given the following 1990 life expectancy table for Americans:

Age	Males	Females
0	70.5	78.1
10	61.7	69.1
20	52.2	59.4
30	43.1	49.7
40	33.8	40.1
50	25.1	30.9
60	17.4	22.4
70	11.2	14.9
80	6.8	8.8

In the year 2010, what would you expect the distribution of ages in the United States to be, given current trends and life expectancy? Determine the impact this might have on our Social Security system, which was designed around 1940.

8. BEARING INSPECTION. On the disk is an ASCII file, **Inspect.Dat**, containing measured diameters from a bearing inspection machine. All measurements are in centimeters. Determine the average and standard deviation. You may want to implement the following formulas to verify that your results are correct.

$$X_i = i^{th} \text{observation} \quad \bar{X} = \frac{\sum X_i}{n}$$

$$S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

9. BASEBALL. Predict the standings of the baseball teams from their team statistics located on disk files called BASEBALL.VAR, BATTER.VAR, and PITCHER.VAR.

10. BICYCLE AXLE. A bicycle axle is designed to be 0.75" in (outside) diameter and to fit into an axle tube 0.76" in (inside) diameter. Samples from the production of axles and tubes are reported in the following table. Is the process working well?

Sample	Axle	Tube
1	0.748	0.745
2	0.753	0.758
3	0.740	0.758
4	0.753	0.765
5	0.751	0.755
6	0.751	0.767
7	0.740	0.768
8	0.736	0.747
9	0.756	0.749
10	0.756	0.755
11	0.751	0.762
12	0.752	0.765

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11. CHAIRS. Chairs are designed for people to sit in.
 - a. What are the most important dimensions in a chair's design?
 - b. How much variability is there in these dimensions in chairs you can measure easily?
 - c. How much variability is there in the corresponding dimensions of people?
 - d. How would you reconcile any differences you found between the dimensions of chairs and of people?
 - e. Are there any published standards for chair dimensions? (Hint: look in the library for a book on architectural graphic standards.)
 - f. How well do the published standards correspond to your observed measurements on chairs and people?
12. POLITICAL DEBATE. Build a table of criteria for assessing a political debate. Take careful notes while watching the debate. Who won? Write a one-page paper justifying your decision within the criteria.
13. WHEN TO TAKE ASPIRIN. The pain reliever market has blossomed in recent years. Today it is virtually impossible to tell which type of over-the-counter medicine is appropriate for a headache, muscle strain, menstrual cramps, toothache, arthritis, or any other type of minor pain these medicines are intended to alleviate. Establish criteria for deciding when each medicine should be taken. Remember to include such items as duration of dosage, severity of pain, and dosage level.
14. POPULATION. Use the data in the following table from the data set in the file POPULATE.VAR on the disk to answer questions a to e. (Source: U.S. Population Statistics: *Information Please Almanac*, 1983)

Year	Pop	Births	Deaths	LT5	A5_19	A20_44	A45_64
1910	92.0	2.8	17.2	11.6	30.4	39.0	14.6
1915		3.0	15.9				
1920	105.7	3.0	13.0	10.9	29.8	38.4	16.1
1925		2.9	11.7				
1930	122.8	2.6	11.3	9.3	29.5	38.3	17.4
1935		2.4	10.9				
1940	131.7	2.6	10.8	8.0	26.4	38.9	19.8
1945		2.9	10.6				
1950	150.7	3.6	9.6	10.7	23.2	37.6	20.3
1955		4.1	9.3				
1960	179.3	4.3	9.5	11.3	27.1	32.2	20.1
1965		3.8	9.4				
1970	203.3	3.7	9.5	8.4	29.5	31.7	20.6
1975		3.1	8.9				
1980	226.5	3.6	8.7	7.2	24.8	37.1	19.6

Note: DEATH represents deaths/100,000; POP and BIRTHS are in millions; other variables are in percentages

- a. Create a new variable OVER65 that represents the percentage of the population that is over 65 years of age.
 - b. Estimate the effect of net immigration in the population figures?
 - c. What is the average growth rate per year? What is your prediction for the population of the U.S. in 1990? How does this compare with a recent estimate for 1989 of 250,410,000?
 - d. Show changes in age composition of the population graphically.
 - e. Are more women of childbearing age having children now than in the past? Are there any noticeable trends in the past? Show the data to support these views.
15. FINAL COURSE EVALUATION. Your instructor's department chair realizes that the student evaluations used do not address the unique features of this class and has asked your class to design a new form. The purposes of the student course evaluations are: to assess your instructor's performance, to assess the appropriateness of the curriculum, and to assess the general class atmosphere. Additional items may be included; however, you are limited to one standard size page.

16. THERMOCOUPLE. When two wires composed of dissimilar metals are welded at both ends and one of the ends is heated, a continuous current flows in a thermoelectric circuit. This is the governing principle behind a temperature measurement device known as the thermocouple. If the other end of the thermocouple is placed in ice water and a voltmeter is inserted in the circuit, the measured voltage is a function of the temperature difference between the ends of the thermocouple. Using the data given below for a type K thermocouple, develop a calibration curve (best-fit line) that you can use to determine the temperature as a function of the measured voltage. In addition, estimate the standard deviation associated with the slope, s_{b1} , and the standard error of the residual s_e . Use the equations given below.

Temperature (C)	Voltage (mv)
0	0.00
20	0.79
40	1.61
60	2.44
80	3.21
100	4.10

Note: Formulas for data analysis:

The sample size = n, the number of valid data points.

The mean:
$$\bar{x} = \frac{\sum x}{n}$$

The sample variance:
$$s_x^2 = \frac{\sum (x - \bar{x})^2}{n - 1} = \frac{\sum (x - \bar{x})(x - \bar{x})}{n - 1}$$

17. MULTIPLE GRAPHS. Identify an application in one of your other classes where it is useful to graph more than one function. Customize your graph with proper axis labels and a title. Write a one page essay discussing your graph and its significance.
18. INVESTMENT. You want to invest \$10,000 for 20 years. A bank offers you the following options for your return on investment.
- $R1 = 10000 * (1+I)^Y$
 - $R2 = 10000 * LN((1+I)^Y)$
 - $R3 = 10000 * 1/(I^Y)$
 - $R4 = 10000 * (3 + (I^Y) + (I^Y)^2)$

where Y = Years, R = Return, and I = Interest. Which of the above arrangements would you like, given that interest rates range from 3% to 20%. Document your solution procedure so that you could convince a friend of the best option.

19. INSTRUCTOR FEEDBACK. Design an assessment system for your instructor to use while assessing your Learning Assessment Journal.
20. SOCIAL POLICY. Establish criteria for assessing published social science or policy-based research. Some issues to consider might include: author bias, corporate sponsorship, PAC involvement, author qualifications, and credibility of the journal.

Select a current issue (such as bottled water versus tap water, or red wine is good for you versus red wine is bad for you, or tobacco companies versus states rights and budgets). Find at least two different articles for each side of the position. Apply your assessment criteria to the journal articles. Based on your criteria, which side is right? Document your thinking process.

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Now look at your criteria. What items need to be changed? Which items were inappropriate? What items need to be added? What conclusions can you make about social science or policy-based research?

21. SWIMMING POOL BACTERIA. The following functional relationships describe the bacterial population in a local swimming pool. If the bacteria count exceeds 500 on two consecutive days the swimming pool must be closed. You are a health official. What factors most concern you and what graphs do you want to monitor?

$$\begin{aligned}
 D &= \text{INDEX}(50) && \text{;Days of summer} \\
 T &= \text{MOVINGAVG}(\text{UNIFORM}(50)*30 + 60,3) && \text{;Temperature} \\
 R &= \text{UNIFORM}(50) && \text{;Rainfall (inches/day)} \\
 S &= T * 10 && \text{;Swimmers a day} \\
 B1 &= \text{EXP}((S-850)/20) * S && \text{;Bacteria1 count} \\
 B2 &= B1/2 + (10+D-\text{ACCSUM}(R))*S/25 && \text{;Bacteria2 count} \\
 B3 &= (B2/B1)/5 && \text{;Bacteria3 count}
 \end{aligned}$$

22. COOLING BEVERAGES. You have just placed two cans of your favorite beverage in the icebox. The temperature of the beverage as a function of time is expected to be:

$$\begin{aligned}
 \text{Temp}(t) &= T_{amb} + (T_{init} - T_{amb}) * \text{EXP}(-t/\tau) \\
 t &= \text{time in minutes} \\
 \text{Temp} &= \text{temperature of the beverage (C)} \\
 \tau &= \text{time constant in minutes} \\
 T_{amb} &= \text{temperature of the icebox (0 C)} \\
 T_{init} &= \text{initial temperature of the beverage (30 C)}
 \end{aligned}$$

Your friend can't wait and pours one of the cans into a glass after 10 minutes. She measures the temperature to be 10 C. How long should you wait to open your can if your ideal drinking temperature is 5 degrees C?

23. TECHNICAL REPORT ASSESSMENT. Develop an assessment form for evaluating a technical report.
24. COMMUTING. Make a table containing the number of miles traveled to campus by each of your classmates as well as their estimated round-trip cost. Classify the modes of transportation used (foot, car, bus, etc.). Through graphical analysis propose an average cost per mile for traveling to campus for all students, as well as for each mode. Clearly outline your solution methodology.
25. THE RENT PROBLEM. Four students are to share a three-bedroom apartment for the school year. The rent is \$700 per month and the apartment is 2200 square feet. The bedroom sizes are 450 square feet, 340 square feet, and 290 square feet. There is a common kitchen area and bathroom. Describe a fair way of determining who gets to choose which room and what share of the total cost each student should pay. Discuss how your solution would change (if at all) if two of the students were brothers who wanted to share a room. Vary the parameters given or add other considerations and discuss their effects.
26. BUYING A CAR. You are planning to buy a car to serve you for four years. Select three makes and models of automobiles you are interested in. For each, determine a realistic price you might expect to pay, equipped as you want it. Determine the expected average miles per gallon for each car for the type of driving you expect to do (estimate the proportions of city vs. highway driving), the expected resale value after four years, expected annual maintenance costs, and insurance costs per year for each car. Making a reasonable estimate of the average price per gallon of gasoline over the next four years, determine which car is the least expensive to own. Do features of either of the other two warrant the additional expense? If you can afford to pay a certain down payment and finance the rest at current rates, how does that affect your solution?
27. GRADE POINT AVERAGE. Create a table which gives department, course title, course number, semester, grade, and credit information for each course you are taking this semester as well as the courses you want to take next semester. Illustrate how you could use information from this table to determine your semester GPA. Test your

calculations to make sure they are correct. Expand this into a system that will tell you what grade you will need in a given course to achieve a desired target GPA, if your other grades are accurately predicted. How many decimal places are GPAs computed to by the registrar's office at your college? How many decimal places is appropriate? Why?

- 28. NATIONAL DEBT. How would you pay off the national debt in five years? What will be the economic effects of your plan? Provide convincing support for each assertion.
- 29. NUTRITION. Determine the current RDA (Recommended Daily Allowance) for a person of your age and gender of each of the following nutrients: Calories, Calories from fat, Total fat, Saturated fat, Cholesterol, Sodium, Total carbohydrates, Dietary fiber, Sugars, Protein, Vitamin A, Vitamin C, Calcium, and Iron (Note that this is NOT a complete list of essential nutrients; you may expand this list.)

Keep a detailed log of everything you eat (including snacks and party food) for one week. Write a report comparing your eating habits with the government's recommendations, enumerating all significant differences. For each such discrepancy, either provide a convincing explanation why it is of no concern, or else suggest a realistic modification of your diet that would lessen the gap.

- 30. TOWERS DESIGN COMPETITION. Your design team is competing for a contract to build transportable radio towers for reporting wildfires in remote locations. It is imperative that these towers be as tall as possible to assure that no fire reports are missed. Using a minimum amount of materials to keep both the cost and weight down is also desirable. The contract will be awarded based on your ability to build a free-standing prototype on the cardboard base you are given. You will be provided with the following materials: (1) Thirty sheets of recycled paper, (2) A roll of transparent tape, (3) A corrugated cardboard base, (4) a pair of scissors, and (5) a calculator. The only materials you may use in your tower are the recycled paper and the transparent tape. Your team's score will be computed by the formula:

$$SCORE = \frac{H}{\sqrt{30 - N}}$$

where H is the height of your tower in centimeters, and N is the number of full, non-mutilated sheets of recycled paper returned at the end of the competition. You will have thirty minutes to choose team roles, develop a plan, build your prototype, and prepare to present an oral report on your team's approach.

- 31. TEMPERATURE CONVERSION. Write a program (in a spreadsheet macro, C, or another programming language) to produce a pair of tables that convert between Celsius and Fahrenheit temperatures. Each table should fit nicely on one printed page.
- 32. MORTGAGE. A fixed interest mortgage normally has a fixed monthly payment which includes all of the current month's interest plus repayment of a portion of the principal due. As more payments are made, the interest part goes down, while the monthly principal repayment goes up. You or your instructor should choose an initial loan amount P, an annual interest rate I, and a number of years of the loan Y (for example, let P = \$50,000, I = 9%, and Y = 20 years). First, determine the correct monthly payment. Because payments are always rounded to the nearest cent, it usually turns out that the correct final payment is a little different from all of the previous monthly payments. Determine the amount F of the final monthly payment for your loan. Along the way, produce a table that shows, for each month, the part of the payment that is repayment of principal, the part that is interest, the total principal repaid to date, and the total interest paid to date. How much does your loan actually cost over its lifetime?
- 33. LOTTERY DESIGN. You are designing a state lottery. But first, you want to look at the experience of other states. In Pennsylvania, while you can select 11 numbers out of 80, 7 of these must be correct to win the grand prize. In New York's lottery, you must choose 6 numbers out of 48 to win. Compare the chances of winning the grand prize in those two states. Your state has a population of 4.5 million and expects to sell 500,000 tickets in an average week. 15% of the proceeds pay for the operation and advertising costs of the lottery, 35% are returned to the cities and towns, and the remaining 50% of the proceeds can be used for prizes. Now design the game so that a winner will occur about every four weeks. Determine the net present value of winning \$50,000 for 20 years (i.e.,

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\$1,000,000 paid off in 20 equal installments).

34. **STOCKS.** Choose ten stocks listed on either the New York or American Stock Exchange to investigate. Determine their performance over the past month, year, five years, and ten years. Considering past performance, volatility, price-earnings ratio, current national and world events, and any other important factors, select three of the ten stocks to “invest” \$50,000 in, split any way you wish between the three stocks. Chart the progress of all ten stocks for the next two months, and then “sell” your stocks. How well did your selection do compared with the other seven? Compared with the Dow? Compared with the S and P 500? Prepare a report, with illustrations, of your experiment and findings. Comment on the pros and cons of investing for a two-month period.
35. **THROWING A BASEBALL.** The trajectory of a baseball in the presence of wind resistance is described by:

$$X = V_i \cos(\theta) (1 - \exp(-k \cdot t)) / k$$

$$Y = (k \cdot V_i \sin(\theta) + g) (1 - \exp(-k \cdot t)) / k^2 - g \cdot t / k + Y_0$$

where

t	=	the time in flight in seconds
k	=	drag (.1/second)
g	=	acceleration due to gravity (9.8 meters/second ²)
theta	=	release angle
V _i	=	release velocity (20 meters/second)
Y ₀	=	release elevation (1.5 meters)

How far can you throw a baseball under these conditions? How should the ball be thrown to give maximum range? Discuss your findings.

36. **TUITION COSTS.** You have just transferred to a new school. You need 60 more credits to graduate and can take 12 to 18 credits each semester. Presently, tuition is \$5000 per semester plus \$100 per credit. How much will it cost you to graduate? During your first semester at the new school, you are told that the semester fee will increase by \$100 each semester from now on and the per credit fee will increase by \$10 each semester. How much will it cost you to graduate on your current schedule? Should you accept the job you have been offered which pays \$1200 per semester but which will require you to take no more than 12 credits each semester?
37. **CHECKBOOK.** Create a software package for balancing your checkbook. Supply a table with six columns as a data structure. The first column of the table should contain the type of transaction (deposit or check), the second should contain the date of the transaction, the third should contain the check number or the deposit number, the fourth should contain the amount of the transaction, the fifth should give the current balance, and the sixth should indicate whether the check has cleared (been processed by the bank). Supply procedures for setting up an initial balance, recording a transaction, making corrections, and displaying one or all transactions with balances. Include validation of all input data and good documentation.
38. **WIDGET PROFITS.** Your factory can produce between one and 25 digital widgets each month. You sell each one for \$40. Materials cost \$10 per widget, and overhead for the part of the factory making widgets is \$50 each month regardless of how many are made. The effect of the union contract is to make it more expensive to make a low or high number of units each month; specifically, labor cost is

$$2 * (\text{units} - 12)^2$$

What is the optimal number of units per month to reduce materials cost? Overhead? Labor cost? Total cost? What is the break-even point where your sales just equal your expenses? How many widgets should you make each month in order to maximize your profits? What happens if competition forces you to lower your selling price by \$5? By \$10?

39. **DIAPERS.** Your next-door neighbors, the environmentalists, have come to you, the problem solver, to help them decide whether to use cloth or disposable diapers on their new baby. You know that expensive studies funded by the makers of each kind of diaper have reached, not surprisingly, opposite conclusions. So there may not be a “right” answer to this problem, only a “best” answer for each particular situation. To help you consistently account

for the various sources of environmental degradation associated with manufacture, transport, use, and disposal of diapers of either type, you have already decided to translate all parameters to dollar costs. Here are suggestions for *some* of the costs you might want to consider:

Rate of use	7 changings per day or 6000 changings per child per lifetime, requiring 6000 disposable or 50 cloth diapers
Purchase price	\$4.95 per cloth diaper, \$0.40 per disposable diaper
Volume occupied	A package of 40 new disposable diapers has the dimensions 7 x 11 x 16 inches; a used disposable diaper occupies 1.5 to 2 times as much volume as a new one
Laundry	15 cloth diapers per load in a washing machine
Water	54 gallons per load at \$1.35 per thousand gallons
Detergent, etc.	\$0.45 per load
Electricity	\$0.55 per load to wash and dry (assuming electricity costs \$0.11 per kilowatt hour)
Sewage	\$2.25 per thousand gallons of sewage
Waste pickup	\$3.50 per week for two 40-gallon bags

Change these values if you think they are not realistic for your area. Add as many other costs as you think necessary, using either determined or assumed values. (If you think that the relatively small dollar amounts involved in this problem make it trivial, remember that about 10 million babies are in diapers at any given time in the United States.)

40. CENSUS. As an employee of the census bureau, you are a member of a team whose responsibility it is to project the nation's population for the year 2000. The team leader suggested that in recent decades growth has been quadratic in nature; that is, given population data from two successive censuses, a quadratic function can model the growth as a function of time. Data for the past five census years are given below:

Year	Population	Rate of Growth (people per year)
1950	151,325,798	2,357,930
1960	179,323,175	2,598,812
1970	203,302,031	2,360,967
1980	226,542,518	2,324,049
1990	249,843,102	2,347,118

Is the team leader's suggestion correct? Either support the quadratic growth hypothesis, or suggest an alternative model. In any event, predict the United States population for the year 2000. How confident are you of your prediction? When actual numbers are available, compare your prediction and comment on any difference.

41. POWER PLANT. Your municipality has decided to build a new gas turbine power plant which has a 20-year lifetime. The construction cost of the plant is anticipated at \$750 per kilowatt of capacity (a 100 kilowatt plant would cost \$75,000). Operating cost is expected to be 4 cents per kilowatt-hour of electrical energy produced and is expected to escalate at 5% per year. The plant will be on line 25% of the time. By law, the municipality can not make a net profit on the plant. How much should each customer pay for each kilowatt-hour of electricity consumed?
42. FURNITURE. Williams' Furniture Crafters recently obtained an order for 18 tables, 72 chairs, and 12 roll-top desks. This work needs to be subcontracted to Woxall Woodcraft, which will charge \$80 per hour for use of its sawing facilities, \$63 per hour for use of the fitting room, \$92 per hour for the finishing facilities, and \$28 an hour

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for quality control inspection.

By experience, Williams' Furniture executives know that it takes 15 minutes to cut the wood for each table, 10 minutes to fit it, 25 minutes to finish it, and 4 minutes for its quality control check. Similarly, each chair requires 25 minutes for cutting, 15 minutes for fitting, 20 minutes for finishing, and 6 minutes for inspection. Desks call for 20 minutes of cutting, 25 minutes for fitting, 25 minutes for finishing, and 7 minutes for inspection.

What Williams' needs to determine is whether Woxall Woodcraft's rates are linearly correlated with the total time that each department is involved in the construction process. If so, then Williams' would like to subcontract the actual shipping of the furniture as well.

If a significant correlation exists, Williams' wants to be prepared to offer a reasonable amount for use of the shipping department, which will require three hours of time. Either recommend what such an offer should be, or else justify your conclusion that Woxall's charges are not particularly well correlated to the time involved.

43. INSULATION. You are going to build a warehouse to store fresh fruits and vegetables for your wholesale produce business in Rapid City, South Dakota. Naturally, you want to spend as little as possible in construction and in heating and cooling to keep your produce at an even 50 degrees F. throughout the year. The warehouse must have at least 20,000 cubic feet of capacity. To minimize operating costs you want to insulate the warehouse to the highest *reasonable* R-value. (Insulation quality is measured by "R-value", where higher values correspond to greater insulating ability.) Walls and ceilings of different R-values do not cost the same. Your research tells you that the *effective* insulative capacity of a square foot of a finished wall or ceiling/roof increases only to a certain point according to the equation

$$R = a - a \cdot \text{EXP}(-b \cdot \text{EXP}(c \cdot \text{COST})),$$

where $a = 50$, $b = 0.02$, and $c = 0.06$. Heat transfer (Q) through a wall or ceiling is measured in BTU/hour, where BTU stands for British Thermal Unit. Heat transfer is given by

$$Q = (SA/R) \cdot (T_{\text{inside}} - T_{\text{outside}}),$$

where SA = surface area of the building in square feet, R = insulative capacity, and T = temperature in degrees Fahrenheit. The average monthly temperatures at your site from January through December are 23, 25, 33, 44, 56, 65, 73, 74, 59, 49, 33, and 25 degrees F respectively. It costs \$0.12 to supply 3413 BTU of electrical energy. Electrical heaters are virtually 100% efficient (i.e., 1 BTU of electrical energy can be translated into 1 BTU of heat). Electric air conditioners are only about 20% efficient (i.e., it takes 5 BTU of energy to remove 1 BTU of heat). Money to purchase the insulative walls and roof/ceiling will be borrowed with interest paid at 8% per year. What insulative capacity will you build into the walls and ceilings of your building?

44. TUNNEL DESIGN. A 548-foot long tunnel is to be built through a nearby ridge in order to accommodate a new highway. The original design calls for the tunnel to have parabolic cross sections and to be 40 feet wide at its base. At a point five feet in from either end of the base, the height of the tunnel must be 12 feet to safely allow truck traffic to pass through. How much dirt must be removed from the ridge to construct this tunnel?

An alternative design would preserve a portion of the parabolic walls as described above, still maintaining the 12-foot height constraint as indicated earlier. But instead of following this parabolic shape all the way, a horizontal roof will be installed that would be 16 feet above the center of the highway. If the alternative design is used, now how much dirt must be removed during construction? Devise a relationship between the cost of dirt removal and the cost of roof construction that could be used to decide which plan is more cost effective.

45. UNIVERSITY CONSTRUCTION. Your university has decided to borrow money at the prime rate to build a 10 unit apartment complex for undergraduate students. If your construction costs are \$40 per square foot, and state law requires the university to break even on this project, determine the rental price for each unit.

46. POKER. Model a poker game: In software (a spreadsheet, or a programming language) create a deck of cards.

Shuffle the deck of cards. Deal a poker hand. Next deal four poker hands. Rearrange the poker hands. Pick up the cards and place them back in the deck.

47. CYLINDRICAL TANK. Determine the least expensive way of building a cylindrical tank which will hold 10,000 cubic feet of gasoline. Assume that metal costs \$5 per square foot and welding costs \$2 per linear foot. Metal is available in sheets up to 8 feet wide by 20 feet long.
48. SCREEN SAVER. Create a screen saving procedure that will draw random patterns on the screen until the user presses Ctrl+Break.
49. SPREADSHEET FUNCTION. Do the same as in the previous problem, but for one of the functions built into your spreadsheet program.
50. CONCEPT MODEL. Develop a computer application for exploring a concept model you learned in this course.
51. CALENDAR. Create a computer application that can produce a calendar showing the dates and days of the week for any given year.
52. DAYS BETWEEN DATES. Create a computer application that does the following: (1) asks the user to enter two dates in MMDDYYYY format (MM = the month, in two digits; DD = the day of the month; YYYY = the year in four digits), (2) computes the number of days there are between the two dates, and (3) displays its results. (NOTE: You will need to account for leap years and leap days. Built-in functions in a spreadsheet may make the task easier.) Be sure to test your application with test dates that cover all the problematic situations that can occur.
53. DESIGN A BETTER SHOPPING CART. Design a better shopping cart for use in supermarkets. Be sure to consider the needs of store owners as well as those of shoppers.
54. PROBLEM IN ANOTHER COURSE. Build a computer application for solving a problem you encounter frequently in one of your other courses.
55. SPELLING. Create a computer application to help youngsters learn how to spell.
56. ARITHMETIC. Create a computer application to help youngsters learn how to do arithmetic (addition, subtraction, multiplication, and division).
57. PREDICTING POPULATION. What will the population of the United States be ten years from now?
58. SPENDING MONEY. (This is a classic problem, first published in 1484 by Nicolas Chuquet of France.) A man spent one-third of his money and lost two-thirds of the remainder. He then had twelve coins left (all of the same denomination). How many coins did he have at first?
59. IRS AUDIT. The IRS audits you. The examiner, who is inexperienced in regards to early loan repayment, is skeptical that your interest payments could be so large that they actually exceed your principal payments in the early stages of a home mortgage. Thus, this improbable situation results in the examiner challenging and disallowing your interest deductions. What do you do?
60. CULTURAL EXCHANGE. You are host coordinator for a workshop being attended by a diverse group of 40 individuals. During the course of the workshop, the performance of one team exceeds that of the other teams. The facilitator congratulates that team's captain with a "high five". The team captain responds to this cultural exchange with a look of "What in the world are you doing?". The facilitator responds with a puzzled question: "Do you know what a 'high five' is?" The foreign born and educated Asian graduate student says "Of course I do". The facilitator says "OK" and continues. Later that week the host coordinator of the workshop gets feedback from this grad student to the effect that the visiting facilitator was a racist and that she wants to file a formal complaint. In the position of host coordinator, what should you do?
61. PAY TO STOP SMOKING. Students in a psychology class have read an article in which excellent results were

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achieved in reduction of smoking by paying smokers according to various schedules including one in which the longer they went without smoking, the more they got paid, up to a specified maximum per week. The students felt this technique was unrealistic because there was no one to pay in ordinary situations. Someone suggested that the money saved could substitute for the money from the research grant that was used in the study. Others questioned the premise that someone could be both the person with the problem and the person who administers the solution. How would you resolve this dilemma?

62. **SHARED HOMEWORK.** Every week you do your homework for a class that is supposed to be done individually by each student. Another student, with whom you are in a group during class time, asks you to share your homework with him the night before it is to be handed in. You feel this is OK because he has also finished the homework on his own and simply wants to compare yours with his. This goes on for a number of weeks. After class one day your teacher asks to see both of you. He shows you your homework assignments for this week and you see that your group member had not just compared your homework to his, as he had done the previous few weeks, but had instead copied yours word for word. He tells the teacher that both of you didn't understand the rules and thought it was OK for both of you to work on the assignment together since you also worked together as a group in class. The teacher asks you if this is true and you are forced into a dilemma: you could tell the truth and say the work was all yours, but that would involve admitting that you still facilitated his cheating by giving him your homework, or you could agree with him and say that you both worked on it equally. You decide to support his story and the teacher accepts the explanation. You begin to feel guilty about lying for someone else's sake, especially because he betrayed your trust. What should you do?
63. **TRAFFIC PLANNING.** We have all encountered intersections where the traffic planners seemed to be completely out of touch with reality. Here's your chance to show that you can do it better. In Newton, Massachusetts, state route 30 (Commonwealth Avenue) runs East and West, and crosses Lowell Ave. (to the North) and Homer Street (to the South). About 50 yards west of this intersection, Valentine Street enters Commonwealth Avenue from the North. The intersections are close enough so that they could arguably be treated as a single intersection. At the least, traffic lights need to be synchronized, if lights are used in both places. Each of the roads in question is wide enough for three lanes (one or two in each direction). Currently, there is a traffic light at the Lowell/Homer/Commonwealth intersection, but not at Valentine Street. Daytime traffic is fairly heavy both directions on Commonwealth Ave., moderately heavy both directions on Homer Street and Lowell Avenue, and fairly light on Valentine Street. At present, traffic heading East on Commonwealth Avenue is often backed up beyond the Valentine Street intersection, so that cars trying to turn left from Valentine Street onto Commonwealth Avenue East have an extremely difficult time. A recent study showed the following percentages of cars going straight ahead or turning from each of the five directions:

Commonwealth Avenue traveling West:

- 10% turn left onto Homer Street,
- 15% turn right onto Lowell Avenue,
- 5% turn right onto Valentine Street, and
- 70% continue West on Commonwealth Ave.

Commonwealth Avenue traveling East:

- 20% turn right onto Homer Street,
- 10% turn left onto Lowell Avenue,
- 5% turn left onto Valentine Street, and
- 65% continue East on Commonwealth Ave.

Homer Street traveling North:

- 35% turn left onto Commonwealth Avenue West,
- 5% turn right onto Commonwealth Avenue East, and
- 60% continue North onto Lowell Avenue.

Lowell Avenue traveling South:

- 20% turn left onto Commonwealth Avenue East,
- 20% turn right onto Commonwealth Avenue West, and

60% continue South onto Homer Street.

Valentine Street traveling South:

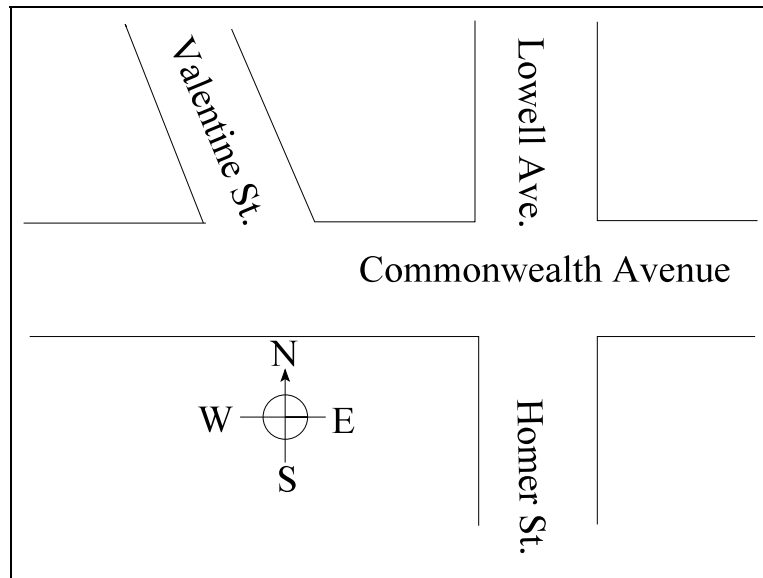
- 70% turn left onto Commonwealth Avenue East, and
- 30% turn right onto Commonwealth Avenue West.

In addition, there is moderate pedestrian and bicycle traffic both East and West on Commonwealth Avenue, and light pedestrian and bicycle traffic in other directions.

Your redesign of this intersection should include answers to the following questions:

- How exactly should lanes be marked (Left or right turn lanes? Combined with straight-ahead lanes?)
- Should there be traffic lights at the Valentine Street intersection?
- How should the traffic light cycles be timed?
- Should some or all of the lights be controlled by magnetic switches in the pavement?
- Should there be a pedestrian cycle (fully automatic, or on demand using a pushbutton)?
- Should any parts of this intersection be signed “No Turn on Red”?

Figure 12.1 -- Intersection.



All persons using this intersection should feel that your resulting design is fair and responsive to their needs. Your writeup of your solution should provide clear justification for every design decision you have made.

64. GUIDELINES FOR CRAPS. Create a set of guidelines for a craps player that will help him/her understand the odds of various tosses occurring given the situations that occur in the game. Use the following rules: On the first roll a 2 or 12 loses and a 7 or 11 wins. On the second role, if you have not won or lost, you win if you match the sum of the first roll or lose if you roll a 2, 7, 11, or 12.
65. DEER POPULATION CONTROL. The logistic equation for population growth is often stated as

$$dx/dt = r * x * (1 - x/k),$$

where r is estimated annual rate of increase of the population, and k is the carrying capacity. The k value is an estimate of the maximum population the environmental conditions can support, and is a function of factors such as an adequate food supply, sufficient ranging area, etc.

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The state Department of Natural Resources has closely monitored the reestablishment of the deer population in one of its state parks. An estimated 15,000 deer now inhabit this park, which can probably maintain only 20,000 deer in all. Left unchecked, the rate of population growth is 5% annually.

A policy must be reached regarding the reinstatement of hunting on these state lands as a “harvesting” process. One of two strategies can be adopted. In the first scenario, the annual harvest can be restricted to a constant number of deer, without taking into account the current population. This alters the logistic equation to

$$dx/dt = r * x * (1 - x/k) - h,$$

where h is the fixed number of deer to be hunted each year.

Alternatively, the harvest rate can be a fixed percent of the existing population; that is, the harvest is proportional to the population size. In this event, the logistic equation is altered to

$$dx/dt = r * x * (1 - x/k) - E * x,$$

where E is the established percent of the population which can be hunted each year.

It is your job to make a recommendation to the Department. Present an argument supporting one of the two plans above, and make a recommendations to the appropriate h or E value.

66. TOWER OF CANS. One of the long-standing sight gags in movies and on television is that of a shopper extracting a single can from a pyramidal tower of cans, only to watch the entire pile come tumbling down. Exactly how many cans are in such a tower?

Assume that the tower has a triangular base, and that each can rests on a triangular support of cans below it (with the obvious exception of the bottom layer). If the top layer consists of a single can, find the total number of cans in a pyramid 20 layers high.

67. MENDELIAN INHERITANCE. Model Mendelian inheritance from heterozygote parents. Assume that each offspring and parent have two genes coding for the characteristic that we are modeling; that of the two genes in each parent one is dominant over the other, that offspring receive one gene from each parent, and that the gene each parent passes down is randomly selected. Examine the state variables as the number of offspring increases.
68. PICNIC WEATHER. It is August 3, and the weather is miserably hot. What makes things worse is that you are hosting a picnic on the 8th and you dread the thought of the weather being hot when your guests arrive. Experience tells you that 60% of the time, an unseasonably hot August day is followed by another hot day; 30% of the time the temperature returns to normal levels; 10% of the time the next day is unseasonably cool. On the other hand, a normal day is followed by a hot day 40% of the time; it remains normal 50% of the time; and 10% of the time unseasonably cool weather follows. Finally, a cool day is likely to be followed by another cool day 20% of the time, by a normal day 60% of the time, and by an unseasonably hot day 20% of the time. How likely is it that the weather will be uncomfortably hot for your picnic?
69. HANGING CABLE. The height of a hanging cable suspended over flat terrain between two high voltage transmission towers is given by:

$$y(x) = H/(2*W) * (EXP(W*X/H) + EXP(-W*X/H)) + y0 - H/W$$

Here H is the horizontal force at location $x = 0$ as well as at $x = a$, where a is the horizontal location of the transmission tower from the origin, $y0$ is the height of the cable above the ground at $x = 0$, and W is the weight of the cable per unit length.

What is the maximum separation between transmission towers if the minimum legal ground clearance is 30 feet, W

= 5 lbs/foot, $H = 1000$ lbs, and the maximum tower height is 100 feet?

What is the maximum allowable weight of the cable per foot if the minimum legal ground clearance is 30 feet, $H = 1000$ lbs, the tower height is 100 feet, and the tower separation is 250 feet?

70. LAB EXPERIMENT SIMULATION. Select a laboratory experiment you performed this semester which involved fitting data to a deterministic model. Assign realistic random errors to the measured variables in your experiment and conduct simulations with your model. Explore the behavior of the state variables in your model. Write a short report which documents your analysis and the significance of your conclusions.

71. COMPONENT FAILURE. The Weibull distribution is a probability density function which gives the probability of failure of a component at a time t after it is manufactured.

$$\text{PDF} = b/\theta * (t/\theta)^{(b - 1)} * \text{EXP}(-((t/\theta)^b))$$

- a) Graph this probability density function for different values of b and θ . Graph the associated cumulative distribution functions. Discuss the roles that b and θ play in this distribution.
- b) Demonstrate how to calculate the probability that a component will fail between 2 and 4 years. In your calculation assume b and θ are both 3 years,
- c) Create a function called SAMPLEWEIBULL which can be used to sample data from the Weibull distribution. Verify that you have implemented this correctly by graphing the frequency distribution resulting from 200 samples.

72. NEW BUSINESS. In the last few years we have seen new small businesses become popular. Examples are video stores, self-service gas stations, quick lubes, and frozen yogurt shops. What new business would you start and why would it be successful?

73. ACREAGE VALUE. How do you determine the value of acreage of the plot of land you want to buy?

74. PET COSTS FOR ONE YEAR. How much does it cost to support a pet for a year?

75. LIFETIME TRANSPORTATION COSTS. How much do you spend on transportation for your lifetime?

76. NEGATIVE INCOME TAX. If we were to implement a negative income tax, what should we set as the minimum amount of money to which every person should be entitled?

77. TEN YEARS SAVINGS. How much money will you have after 10 years if you save 100 dollars each month?

78. RECYCLING ECONOMICS. What economic system would you put in place to guarantee a 75% recycling of aluminum, glass, and paper?

79. GOLF. If your tee shot on a par-3 hole lands on the green, what is the probability that it will fall into the cup?

80. COOKING LASAGNA. How do you cook lasagna?

81. NBA STRATEGY. Should the National Basketball Association implement a pay-per-view, local cable, or network strategy for the next decade? Prepare a paper to propose a strategy for the owners.

82. VALUE OF GOVERNMENT SERVICES. If you were to accept a check from all the levels of government instead of all services provided by those levels, how much should the check be?

83. ELEVATOR TIMING. The Sawyer building at Suffolk University has five elevators and twelve floors. When class times change, there is always a large crowd on the first floor, so people wanting to get on from the second or

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third floors to go up often have to wait for several elevators to go by before they can get on. Devise a scheme for controlling the elevators that will address in a fair manner the needs of people starting from the first or higher floors.

84. AVERAGE TAXPAYER. What does the average taxpayer pay in taxes as a percentage of income, given the data below?

Figure 12.2 -- Tax Table

Tax Table	
Income	Tax Rate
0 - \$ 9,000	0%
\$ 9,001 - \$14,000	15%
\$14,001 - \$22,000	20%
\$22,001 - \$45,000	28%
\$45,001 - \$64,000	33%
\$64,001 +	38%

Figure 12.3 -- Wage Earner Distribution

Wage Earner Distribution	
% of Employees	Income (at least)
15%	\$15,000
7%	\$20,000
9%	\$25,000
21%	\$30,000
13%	\$35,000
6%	\$40,000
4%	\$45,000
8%	\$55,000
5%	\$75,000
6%	\$90,000
2%	\$125,000
4%	\$150,000

85. 30K PER YEAR. What can you do with \$30,000 per year in salary?
86. COSTS OF HIGHER EDUCATION. Pick three colleges: a community college, a public four-year college, and a private four-year college. Which type of education is the best buy? For whom?
87. GETTING A JOB. What process would you use to get a job that would earn you \$8,000 per year for the nine months while you go to school?
88. POINTS PER POSSESSION. Say you've made a study of the game of professional basketball, and have determined that the key model for effective basketball involves *points per possession*. What contribution would each of the following make to the model: field goal percentage; free throw attempts; free throw percentage; turnovers; offensive rebounds; defensive rebounds; steals?
89. SAVINGS COMPARISON. Having just graduated from college, at age 22, you have two job offers with companies, either of which you would be happy to work for during your entire working career. Both are offering the same salary and benefits with one difference: Company A offers to contribute \$100 every month toward your retirement fund, investing the money in a mutual fund that has historically earned 12% every year; while Company B offers to contribute \$250 every month, investing it in the same mutual fund. The hitch is that Company B won't start making these contributions until you have worked for them for ten years. If you worked for either company until age 65, how much would you have in your retirement fund? Which is the better job offer? How much should Company B be willing to contribute in order for your accumulated total to be exactly the same, at age 65, as for Company A?
90. COLLEGE BUDGET. Obtain a copy of last year's budget for your school. Discuss the major expenses as well as the major sources of money. Look at the cash flow situation. Make suggestions where costs could be cut and where money should be spent where it currently is not.
91. WEB PAGE. Read up a bit about HTML, and design a personal web page for the World Wide Web. Be sure to include some attractive formatting features, graphics, animation, and links to other web pages.
92. WHAT'S IN A NAME? What is the origin of the name "Myrvaagnes", and what does it mean?
93. COVER PICTURE. Figure out what the cover picture has to do with the five main themes of this book. Find as many connections as you can, and explain them clearly.
94. AIRCRAFT CRASH I. When an aircraft crashes, National Transportation Safety Board (NTSB) or military investigators need to try to reconstruct the accident to determine what happened. This problem and the next two give examples of some of the problems investigators must solve.
- An aircraft crashes in a level open field. The flight path angle is 10 degrees and true airspeed is 85 miles per hour. Initial impact occurs with the fuselage level (zero pitch angle). The impact causes a two foot deep gouge, and the aircraft comes to rest 25 feet from initial impact. The fuselage is crushed 12 inches vertically and 5 feet horizontally.
- FIND:
1. Aircraft ground speed (V_h) and vertical velocity (V_v) in feet per second.
 2. The mean vertical and horizontal accelerations, in G's.
 3. The magnitude and direction of the mean crash resultant.
95. AIRCRAFT CRASH II. An airplane crashes on level ground at an airspeed of 140 knots. Accident investigators discover that the airplane struck the top of a tree at a point 60 feet above the ground and crashed 100 feet from the base of the tree. The aircraft came to rest at the end of a gouge 32 feet long and 3 feet deep. Measurements show that the airplane was crushed 60 inches longitudinally and 12 inches vertically.
- FIND:
1. Horizontal and vertical velocities, in feet per second.
 2. Mean horizontal and vertical accelerations, in G's.
 3. Magnitude and direction of crash force resultant.

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96. AIRCRAFT CRASH III. An airplane crashes against a 10 degree uphill slope. The impact angle is 20 degrees. At the time of impact the aircraft vertical velocity was 1800 feet per minute. The airplane came to rest after sliding 80 feet. Maximum depth of the gouge was one foot, and inspection revealed that the airplane structure was crushed one foot vertically. There was no horizontal crushing of structure.

FIND:

1. Flight path velocity.
2. Mean longitudinal and vertical accelerations, in G s, with respect to the face of the hill.
3. Mean crash force resultant magnitude and direction.

97. AIRCRAFT CRASH IV. In the accident in problem 96, the longitudinal axis of the airplane is parallel to the flight path.

FIND:

Longitudinal and vertical accelerations with respect to the aircraft axes.

98. RETIRE IN TEN YEARS. A financial planner has proposed that anyone who invests wisely 10% of their income each year can expect to retire in ten years. What assumptions do you think lie behind this assertion? Come up with a detailed plan for how you might retire ten years after graduating from college. What would be your lifestyle if you did that?

99. IMPROVE TEACHING. Many newspaper and magazine articles over the past several years have bemoaned the fact that bright young people are not attracted to teaching as a profession, and that as a result there will be a severe shortage of qualified teachers in a few more years. Devise a plan to correct this situation. And devise a plan to sell your plan politically.

100. CORE CURRICULUM. Study the core curriculum at your college (the core curriculum is the set of courses that all students must take, and the set of requirements that all must pass). Determine the historic reason for each current requirement. Then propose a new plan that will better fit the needs of students of the twenty-first century.